

Module 5: Algebra

VARIABLES

Variables are unknown quantities. Letters are used to represent variables.

ALGEBRAIC EXPRESSIONS

Algebraic expressions are made up of **terms**.

Examples of algebraic terms are x , $2y$, m^2 , 5 , $\frac{x}{y}$, mn .

The term $2y$ has a co-efficient of 2 and y is the variable. In the term $-8xy$, -8 is the co-efficient and xy is the variable part of the term.

A term may be a constant, such as 5.

The following terms have been expanded to show the operations, multiplication is implied when two or more symbols are written side by side.

$$2y = 2 \times y$$

$$m^2 = m \times m$$

$$mn = m \times n$$

$$\frac{x}{y} = x \div y$$

Exercise 1:

Substitute $a = 2$ and $b = -3$ into the following terms to find its value.

a. $5a =$

d. $(3b)^2 =$

b. $3b^2 =$

e. $a b^2 =$

c. $b^2a =$

f. $\frac{1}{5a^2} =$

Like terms are terms with the same variable. Examples of like terms are, $3x$ and $4x$, $5x^2$ and x^2 , $-6x$ and $5x$, xy^3 and $4xy^3$, $-8y^3$ and $3y^3$. The coefficients of like terms may be added or subtracted from each other. For example:

$$3x + 4x = 7x \quad , \quad 6y^2 - 4y^2 = 2y^2$$

Exercise 2

Simplify the following algebraic expressions by adding or subtracting the like terms. In all cases, the expression will simplify to one term.

a. $3 + 10 =$

b. $x + x =$

c. $3x + 4x =$

d. $-6x + 5x =$

e. $5x^2 + x^2 =$

f. $xy^3 + xy^3 =$

g. $-8y^3 + 3y^3 =$

h. $3 - 10 =$

i. $x - x =$

j. $3x - 4x =$

k. $-6x - 5x =$

l. $5x^2 - x^2 =$

m. $xy^3 - 4xy^3 =$

n. $-8y^3 - 3y^3 =$

Algebraic terms may be multiplied together and then simplified into a single term. For example:

(i) $7a \times 0.5a^2$ means $7 \times a \times 0.5 \times a \times a$ which is the same as $3.5a^3$

(ii) $-6a \times 2b$ means $-6 \times a \times 2 \times b$ which is the same as $-12ab$

Exercise 3

Simplify the following algebraic expressions by multiplying the given terms.

a. $2 \times 5a =$

b. $6y \times 5y =$

c. $3a \times 2a =$

d. $-2m \times 7n =$

e. $10y \times 0.5 \times 3y =$

f. $-3m \times -2 \times m =$

g. $4z^2 \times 6z =$

h. $-6d \times -\frac{1}{2}d =$

Algebraic terms may also be divided and simplified, if possible.

For example: $a^2 \div a = \frac{a \times a}{a} = a$

$$10b^2 \div 5b = \frac{10b^2}{5b} = \frac{10 \times b \times b}{5 \times b} = 2b$$

Exercise 4

Simplify the following algebraic expressions by dividing the given term. Rewrite the expression as an algebraic fraction first, and then simplify as much as possible.

a. $6y \div 2 = \frac{6y}{2} =$

b. $12x^2 \div 6x^2 =$

c. $-6yz \div 2zy =$

d. $-21ab \div (-3a) =$

e. $z^2 \div (-2z) =$

f. $4abc \div 3abc =$

Algebraic expressions like $4(p + 2)$ mean 4 'lots of' $(p + 2)$ which is $4p+8$ when expanded. That is, $(p+2)+(p+2)+(p+2)+(p+2) = 4p + 8$.

A method to **remove the brackets** from similar expressions is demonstrated below. Multiply the term outside the bracket, by each term within the bracket.

For example: (i) $3(9x + 2) = (3)(9x) + (3)(2) = 27x + 6$

(ii) $-3(9x + 2) = (-3)(9x) + (-3)(2) = -27x - 6$

(iii) $3(9x - 2) = (3)(9x) - (3)(2) = 27x - 6$

(iv) $-3(-9x - 2) = (-3)(-9x) - (-3)(2) = 27x + 6$.

Exercise 5

Expand the following expressions by removing the brackets.

a. $3(y + 2) =$

b. $4(y - 6) =$

c. $-2(x + 9) =$

In part d) and e) below, to remove the bracket, multiply the term outside the bracket with each term within the bracket.

d. $m(m + 2) =$

e. $3y(xy + y + 2x) =$

In part f) and g) below there is a negative sign outside the bracket. To remove the bracket, each term within the bracket must be multiplied by -1 .

f. $-(2 - 7x) =$

g. $-(z - 2) =$

Exercise 6

First remove the bracket, then simplify each expression by adding or subtracting the like terms.

a. $2(z + 3) + 5 - 2z + 7 =$

b. $3x + 5 + 5(2x + 4) - 2x + 6 =$

c. $3(x - 1) + 4(x + 2) =$

EQUATIONS

An equation is algebraic expression which contains an equality sign; there may be one or more algebraic terms on either side of the equal sign. The left hand side (LHS) of the equation equals the right hand side (RHS) of the equation. Equations can be rearranged and simplified provided the same operation is carried out **on both sides** of the equation; hence the LHS will remain equivalent to the RHS.

Adding or subtracting the same term to both sides of an equation.

Worked examples

Solve the following equations for x:

i. $x+9=16$ Subtract 9 from both sides of the equation

$$x+9-9=16-9 \quad \text{Note: } +9-9=0$$

$$x=7$$

ii. $x-4=10$ Add 4 to both sides of the equation

$$x-4+4=10+4 \quad \text{Note: } -4+4=0$$

$$x=14$$

Multiplying or dividing both sides of a given equation by the same term.

Worked examples

Solve the following equations for x:

i. $3x = 6$ Divide both sides by 3. (The coefficient on the x.)

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2$$

ii. $-7x = 28$ Divide both sides by -7. (The coefficient on the x.)

$$\frac{-7x}{-7} = \frac{28}{-7}$$

$$x = -4$$

iii. $\frac{x}{3} = -8$ Multiply both sides by 3

$$\frac{x}{3} \times 3 = -8 \times 3 \dots\dots\dots 3 \text{ divided by } 3 \text{ on the LHS gives } 1$$

$$\boxed{x = -24} \dots\dots\dots$$

iv. $\frac{14}{6} = \frac{7}{x}$

$$\frac{6}{14} = \frac{x}{7} \quad \text{Reciprocate the fractions}$$

$$\frac{6}{14} \times 7 = \frac{x}{7} \times 7 \quad \text{Multiply both sides by } 7, \text{ one } x \text{ is left on the RHS}$$

$$3 = x \quad \text{or} \quad \boxed{x = 3}$$

v. $3x+5 = x+9$ Subtract x from both sides of the equation

$$3x-x+5 = x-x+9 \quad \text{Note: } x-x=0$$

$$2x+5 = 9$$

$$2x+5 - 5 = 9 - 5 \quad \text{Subtract } 5 \text{ from both sides of the equation}$$

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2} \quad \text{Divide both sides by } 2$$

$$\boxed{x = 2}$$

vi. $56 = 7(2+0.1x)$ Divide both sides by 7

$$\frac{56}{7} = \frac{7(2+0.1x)}{7}$$

$$8 = 2+0.1x \quad \text{Subtract } 2 \text{ from both sides}$$

$$8 - 2 = 2 - 2 + 0.1x$$

$$6 = 0.1x \quad \text{divide both sides by } 0.1 \text{ and we are left with } \boxed{x=60}$$

Exercise 7

Solve the following equations for x.

a) $x+7=3$	b) $8+x=-5$
c) $2x+3=8$	d) $4+5x=20$
e) $3x+8=8$	f) $3x+8=2x$
g) $2x+6=3x+9$	h) $4-x=9$
i) $3(x+2)=2(x+8)$	j) $4x-7=3x-4$
k) $2x+6=3x-9$	l) $3(x-4)=5(x-3)$
m) $2+2x+3=7(x-5)$	n) $-10x+11=4x-17$
o) $\frac{x}{5} = 11$	p) $14 = -\frac{x}{2}$
q) $\frac{8x+4}{2} = 1$	r) $\frac{6x}{3} + 6x = 16$

ANSWERS TO EXERCISES

ALGEBRAIC EXPRESSIONS

Exercise 1

a. $5 \times 2 = 10$

c. $(-3) \times (-3) \times 2 = 18$

e. $2 \times (-3) \times (-3) = 18$

b. $3 \times (-3) \times (-3) = 27$

d. $(3 \times (-3))^2 = (-9)^2$
 $= -9 \times -9 = 81$

f. $\frac{1}{5 \times 2^2} = \frac{1}{20} = 0.05$

Exercise 2

a. $3 + 10 = 13$

f. $xy^3 + xy^3 = 2xy^3$

k. $-6x - 5x = -11x$

b. $x + x = 2x$

g. $-8y^3 + 3y^3 = -5y^3$

l. $5x^2 - x^2 = 4x^2$

c. $3x + 4x = 7x$

h. $3 - 10 = -7$

m. $y^3x - 4xy^3 = -3xy^3$

d. $-6x + 5x = -x$

i. $x - x = 0$

n. $-8y^3 - 3y^3 = -11y^3$

e. $5x^2 + x^2 = 6x^2$

j. $3x - 4x = -x$

Exercise 3

a. $10a$

d. $-14mn$

g. $24z^3$

b. $30y^2$

e. $15y^2$

h. $3d^2$

c. $6a^2$

f. $6m^2$

Exercise 4

a. $3y$

d. $7b$

b. 2

e. $-0.5z$

c. -3

f. $\frac{4}{3}$

Exercise 5

a. $3(y+2) = 3y+6$

c. $-2(x+9) = -2x-18$

f. $-(2-7x) = -2+7x$

b. $4(y-6) = 4y-24$

d. $m(m+2) = m^2+2m$

g. $-(z-2) = -z+2$

e. $3y(xy+y+2x) = 3xy^2+3y^2+6xy$

Exercise 6

a. $2(z+3)+5-2z+7$
 $= 2z+6+12-2z$
 $= 18$

b. $3x+5+5(2x+4)-2x+6$
 $= x+11+10x+20$
 $= 11x+31$

c. $3(x-1)+4(x+2)$
 $= 3x-3+4x+8$
 $= 7x+5$

EQUATIONS

Exercise 7

a. $x+7-7=3-7$
 $x=-4$

b. $8-8+x=-5-8$
 $x=-13$

c. $2x+3-3=8-3$
 $2x=5$

$$\frac{2x}{2} = \frac{5}{2}$$

$x=2.5$

d. $4+5x-4=20-4$
 $5x=16$
 $\frac{5x}{5} = \frac{16}{5}$

$$\frac{5}{5} = \frac{16}{5}$$

$x=3.2$

e. $3x+8-8=8-8$
 $3x=0$
 $\frac{3x}{3} = \frac{0}{3}$
 $x=0$

f. $3x+8-2x=2x-2x$
 $x+8=0$
 $x+8-8=0-8$
 $x=-8$

g. $2x+6-2x=3x+9-2x$
 $6=x+9$
 $-3=x$
 $x=-3$

h. $4-x+x=9+x$
 $4=9+x$
 $4-9=9-9+x$
 $-5=x$
 $x=-5$

i. $3x+6=2x+16$
 $3x-2x=16-6$
 $x=10$

j. $4x-3x-7=3x-3x-4$
 $x-7=-4$
 $x-7+7=-4+7$
 $x=3$

k. $2x-2x+6=3x-2x-9$
 $6=x-9$
 $15=x$
 $x=15$

l. $3x-12=5x-15$
 $-12+15=5x-3x$
 $3=2x$
 $x=1.5$

m. $2+2x+3=7(x-5)$
 $5+2x=7x-35$
 $40=5x$
 $x=8$

n. $-10x+11=4x-17$
 $11+17=4x+10x$
 $28=14x$
 $\frac{28}{14} = \frac{14}{14}x$
 $x=2$

o. $\frac{x}{5} \times 5 = 11 \times 5$
 $x=55$

p. $14 = -\frac{x}{2} \times 2$
 $2 \times 14 = -x$
 $-x=28$
 $x=-28$

q. $\frac{8x+4}{2} \times 2 = 1 \times 2$
 $8x+4=2$
 $8x=-2$
 $\frac{-2}{8} = \frac{1}{4}$

r. $\frac{6x}{3} + 6x = 16$

$$2x+6x=16$$
$$8x=16$$

$x=2$